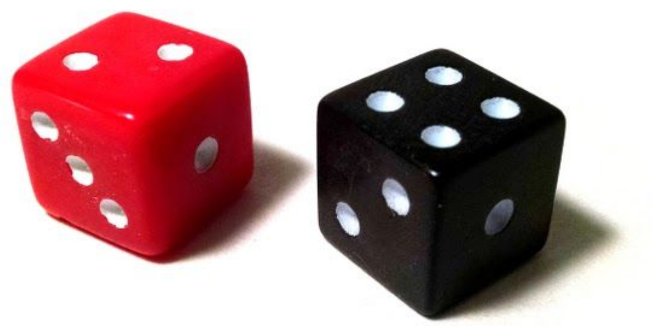
The Unfair Dice Problem (Simulation)

Laura Gutierrez Funderburk, Richard Hoshino, Michael Lamoureux



Activity summary

The unfair dice problem is a free resource for teachers and students, and is part of the Callysto project, a federally-funded initiative to bring data science skills into Grade 5-12 Canadian classrooms.

In this activity students will use an online Callysto notebook, which simulates an unfair dice game. This can be done with students in person, or, online.

Grade level

This activity is best suited for students in Grades 9 to 12, but can be modified for students in Grades 5 to 8.

Learning outcomes

- Probabilities
- Problem solving
- Data analysis

Required materials

1. A charged computer.

2. Access to the internet.
3. Preferably Google Chrome, Safari or Firefox installed.

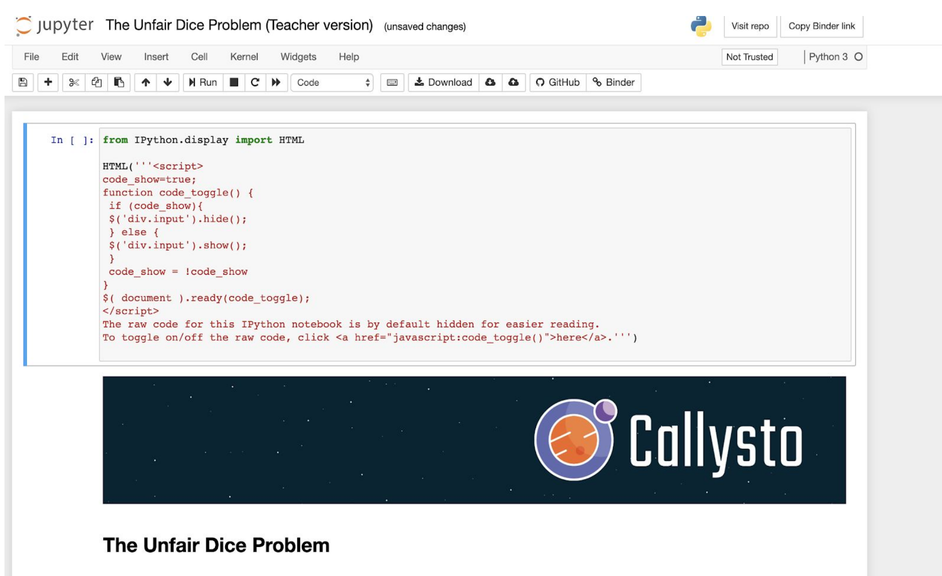
Preparation

1. Visit this [link](). It contains a student and teacher version of this online notebook. Both notebooks contain the same exercises, but the teacher version contains additional notes, suggested discussion items, as well as answers to the student exercises.

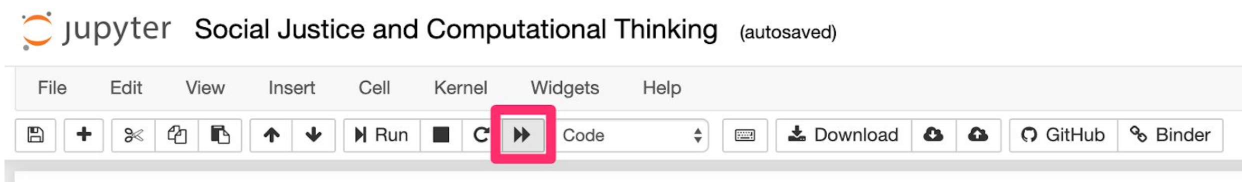
2. This link will take you to a screen that looks like this:

3. Students click “Student Notebook” and teachers click “Teacher Notebook”.

4. When opening both notebooks, the top of the page looks as follows:



5. To start the notebook, press the >> button. Confirm “Restart and Run All Cells”.



During the activity:

1. Show the students the sample space of the game, the probability associated with each player winning, as well as the expected per-round payoff. All of this information is in the notebook.

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	-1	0	1	2	3	4
3	-2	-1	0	1	2	3
4	-3	-2	-1	0	1	2
5	-4	-3	-2	-1	0	1
6	-5	-4	-3	-2	-1	0

Ask the students to use the diagram below to determine what the probability each player will win is.
The probability Alice will win is

$$P(A) = \frac{21}{36}$$

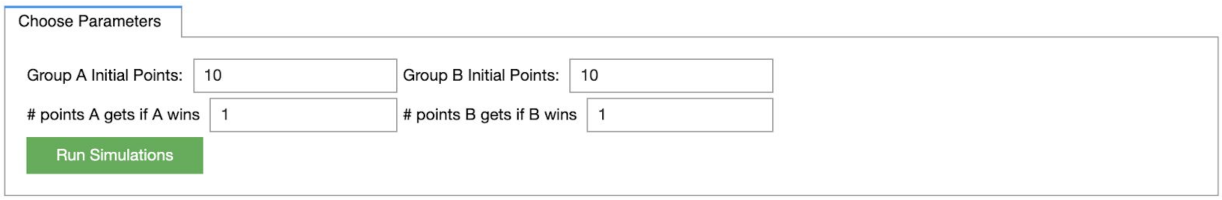
while the probability Bob will win is

$$P(B) = \frac{15}{36}$$

2. Compare with the students' answers from the hands-on discussions.

3. An interesting exercise is to change the number of points to make the game as fair as possible. This happens when both players win approximately 50% of the time - the simulation will provide this feedback. This happens exactly when the expected per round

payoff equation is equal to zero. Ask the students to
change the number of points, and investigate how it changes the game.



4. While the students play with the notebook, and once they have found a solution, ask them to take note of:

    a. Group A initial points in your solution.
    b. Group B initial points in your solution.
    c. Number of points Group A gets if they win.
    d. Number of points Group B gets if they win.
    e. The average number of rounds it took for the game to end. (The game ends when one of the two players does not have any points left).
    f. If your solution involves changing a different aspect of the game other than the number of points, describe what you changed.

They can enter their solutions and save them in the student version. Create room for discussion and have the students share their solutions with the class and how they found them. Some students may attempt to solve the equation

P(A)x Number of points Alice gets - P(B)x Number of points Bob gets = 0

Where:
P(A) = probability Alice will win and P(B) = probability Bob will win.

Other students might find it while tinkering with the simulation tool. There are multiple solutions, some of which can be encapsulated in the following table:

Initial Points A	Initial Points B	Points A gets if A wins	Points B gets if B wins
100	100	5 (or 10, 15, 20, 25,...)	7 (resp 14, 21, 28, 35,...)
10*	10*	1 (or 2,3,4,5,...)	1.4 (resp 2.8, 4.2, 5.6, 7,...)

* The number of initial points can change and will not impact the expected payoff result, but it will impact how long it takes for the game to end.

Suggested discussion questions

Ask students to brainstorm situations where one person has an advantage for another, and, what solutions could balance the outcome. This is also a problem-solving exercise.